# APPLE MAGNETIC ATTITUDE CONTROL (AMAC) — A NEW ATTITUDE CONTROL CONCEPT

**P. S. Goel and S. Rajaram**

*ISRO Satellite Centre, Bangalore-560 OS8, India*

ABSTRACT

A novel closed loop magnetic attitude control scheme for momentum biased, near equatorial orbit satellites is described. Unlike the other schemes proposed so far, the controller presented here performs both the attitude correction and nutation damping, thus obviating the need for a seperate nutation damping mechanism.

The magnetic torquer is placed along the roll axis of the spacecraft. The roll error, $\phi$, obtained from earth sensor, is filtered out into two components viz., one varying at orbital frequency, $\phi_o$, and the other varying at nutational frequency $\phi_n$. The control dipole moment, Mc, of the magneto-torquer is governed by the control law, $M_c = K_2'\phi_n - K_1'\phi_o$ where $K_1' K_2'$ are constants.

Analytical expressions for time response of the system and conditions for stability are derived, using linearised equations of motion. The roll/yaw dynamics of the satellite was simulated on an analog computer and the simulation and analytical results matched well. Also, the simulation results indicate that enough damping is provided in the yaw channel; design aspects such as choice of feedback gains and saturation characteristics of the magneto-torquer are discussed.

Key Words : Magnetic attitude control, Body stabilised satellites.

## 1.0 INTRODUCTION

The current trend in Satellite attitude control shows that 3-axis stabilisation with momentum biased wheels will be adequate to meet the stringent pointing requirements of present and future communication satellites. The need for long life communication satellites has led to the development of Magnetically Suspended Momentum Wheel(MSMW), having an operational life of fifteen years. However, the use of the chemical thrusters for Roll/Yaw control for such a long time becomes questionable due to the increased number of thruster firings and propellant weight and associated contamination problems. Electrical thrusters, though gaining importance, do not have a demonstrated life of more than two years. Hence there is a need for the evolution of semi-passive controllers utilising the environmental forces such as earth's magnetic field, solar radiation pressure etc.

The utility of magnetic torquers for satellite control has been well established for near earth orbits(Ref.1-5) The extension of magnetic torquers for satellite in geosynchronous altitude is receiving considerable attention due to the successful demonstration in the Lincoln Experimental Satellite 5(LES 5) and RCA's SATCOM(Refs.6-7). Recently, Lacombe analysed the feasibility of magnetic torquing in the case of a single degree freedom angular momentum type control of a large satellite.

The present work investigates a closed loop magnetic attitude control system for roll/yaw control of a momentum biased body stabilised APPLE Spacecraft, the first Indian experimental communication satellite to be launched in the middle of 1980 by the ARIANE Launch Vehicle. The proposed controller corrects the attitude error and also damps out nutational oscillations, arising due to transverse torquing, thus eliminating the need for WHECON[9] or half precession cycle damping by

reaction jets. The yaw control is obtained by roll/yaw coupling established by the momentum wheel and is quite adequate for most of the applications. An additional feature of the control system is that it can provide long term attitude stability such as those required by certain meteorological missions(.004°/30 minutes), which otherwise cannot be met by momentum wheels alone.

The analysis design and simulation results of the proposed control scheme are discussed in the subsequent sections. The implementation of the control law is also covered.

## 2.0 GEOMAGNETIC FIELD AT SYNCHRONOUS ALTITUDE

Extensive observations of the geomagnetic field by several satellites now provide a realistic picture of the earth's magnetic field configuration. The magnitude of the geomagnetic field at any satellite station or longitude can be extrapolated from the observations made by ATS1 and ATS 6 satellites. The equatorial magnetic field at the geocentric distance of about 6.55 times earth's radius consists of a steady state geomagnetic field on which temporal variations are superimposed. On magnetically quiet days, the geomagnetic field is along the orbit normal pointing southwards. External field sources cause a diurnal variation of 10 to 15 per cent in the total magnetic intensity. During magnetic storms, the field at synchronous altitude can increase by 50 per cent, decrease to zero or even reverse in its direction; such intense magnetic storms may occur about a maximum of four times, depending upon solar activity. In general the effects of storms do not change significantly the pitch component of the magnetic induction in the 6-18 hours satellite local time[8]. The storms usually occur about noon time. The recovery of the geomagnetic field after a severe storm may require several days.

It is assumed here that the pitch component of the magnetic field is about 1 milli gauss. Although the inplane component of the magnetic field may vary from 0 to 40 gammas depending upon the satellite location, it is assumed to be zero, as the error caused by it will be taken care of by the pitch control system.

## 3.0 EQUATIONS OF MOTION

Let XYZ represent a reference co-ordinate system with the origin at the centre of mass of the satellite as shown in fig.1. The principal axes of the spacecraft x,y and z are to be nominally aligned with the reference axes X,Y and Z respectively. The orientation of the body fixed axes with respect to the reference co-ordinate system can beobtained through the standard yaw ($\Psi$), roll ($\Phi$) and pitch ($\Theta$) angles.

The linearised equations of motion are given as

$$\ddot{\Psi} + a_1\Psi + b_1\dot{\Phi} = Tx/Ix \quad .. \quad (1)$$

$$\ddot{\Phi} + a_2\Phi + b_2\dot{\Psi} = Ty/Iy \quad .. \quad (2)$$

where,

$$a_1 = -w_o(h + w_o I_y)/Ix \quad .. \quad (3)$$

$$a_2 = -w_o(h+w_o I_x)/Iy \quad .. \quad (4)$$

$$b_1 = \left[h+w_o(Ix + Iy)\right]/Ix \quad .. \quad (5)$$

$$b_2 = -\left[h + w_o(Ix+Iy)\right]/Iy \quad .. \quad (6)$$

$h = h-w_o I_z$

$I_x$, $I_y$, $I_z$ = the moments of inertia about x,y and z axes.

$w_o$ = orbital velosity

$T_x$, $T_y$ = Torques

Laplace transformation of the linearised equations yields

$$\begin{bmatrix} (s^2+a_1) & b_1 s \\ b_2 s & (s^2+a_2) \end{bmatrix} \begin{bmatrix} \Psi(s) \\ \Phi(s) \end{bmatrix} =$$

$$\begin{bmatrix} \frac{T_x(s)}{I_x} + s\Psi_o + \dot{\Psi}_o + b_1\Phi_o \\ \frac{T_y(s)}{I_y} + s\Phi_o + \dot{\Phi}_o + b_2\Psi_o \end{bmatrix} \quad .. \quad (7)$$

where,

$\Psi_o$ - Initial yaw angle

$\Phi_o$ - Initial roll angle

$\dot{\Psi}_o$ - Initial Yaw rate

$\dot{\Phi}_o$ - Initial roll rate.

The characteristic equation of the system is given by;

$$(s^2+a_1)(s^2+a_2)-s^2 b_1 b_2 = 0 \quad .. \quad (8)$$

The roots of the above characteristic equation indicate that the system has two natural frequencies

viz.,
(1) the nutation frequency, $w_n = h/\sqrt{I_x I_y}$ and

(2) the orbital frequency, $w_o$. If there is no nutation, an error in yaw gets converted to a roll error after quarter of an orbit. The problem now is to determine a control law so that nutation and the orbital frequency components are damped out i.e., the momentum wheel axes coincides with total angular momentum vector and also the momentum vector aligns with the orbit normal.

## 4.0 PROPORTIONAL CONTROLLER

Let us now consider an electromagnet placed along the roll axis of the satellite. Also, the dipole moment of the magnet is varied proportional to the roll error. The torque produced by the interaction of the magnetotorquer with the geomagnetic field is given as

$$\vec{T} = \vec{M} \times \vec{B}_E \qquad (9)$$

where,

M = dipole moment of the magnetotorquer.

$B_E$ = Geomagnetic field.

Since the magnitude of $B_E$ is fairly constant, a yaw torque proportional to roll error is obtained,

i.e., $T_x = K'\phi(t)$

Where K' is a feedback constant.

The new characteristic equation is given as

$$(s^2+a_1)(s^2+a_2)-s^2b_1b_2+Kb_2s = 0 \qquad (10)$$

Equation (10) can be rewritten as

$$(s^2+\lambda_1 s+w_1^2)(s^2+\lambda_2 s+w_2^2) = 0 \qquad (11)$$

It can be shown that

$$\lambda_1 = -Kb_2/w_2^2 \qquad (12)$$

and $$\lambda_2 = +Kb_2/w_2^2 \qquad (13)$$

The equations (12) and (13) show that a proportional controller with a positive value of K will provide damping of the nutational motion whereas the orbital frequency component would increase. On the other hand a negative value of K reduces the amplitude of the low frequency component but causes nutational instability. Thus the dual objectives of error reduction and nutation damping cannot be simultaneously

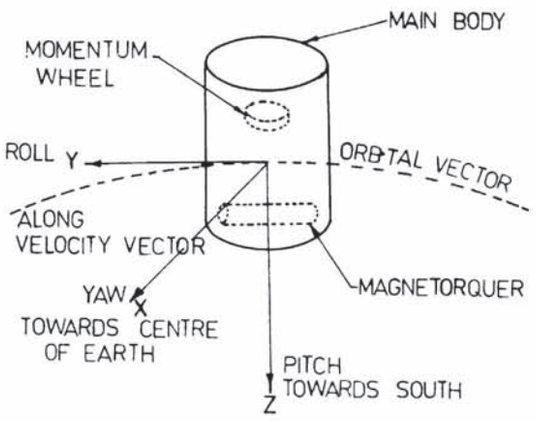



Fig.1 SATELLITE CONFIGURATION achieved by the proportional controller alone.

## 5.0 PROPOSED CONTROLLER

In this section a new control law is proposed such that the dipole moment of the magnetotorquer and hence the torque developed are governed by the following relationship.

$$Mc = -K_1'\phi_o+K_2'\phi_n \qquad (14)$$

and $$T_x = -K_1''\phi_o+K_2''\phi_n \qquad (15)$$

Where,

$\phi_o$ - Orbital frequency component of the Earth sensor output.

$\phi_n$ - Nutational frequency component of the Earth sensor output.

$K_1', K_2'$ - Feedback gains.

The schematic diagram of the proposed controller is given in fig.2. The signals $\phi_o$ and $\phi_n$ can be obtained by passing the output of the Earth sensor through a Low Pass Filter(LPF) and a High Pass Filter (HPF) respectively. The transfer function of the LPF is given by,

$$\phi_o(s) = \phi(s)/(1+\tau s) \qquad (16)$$

Also, the output of the HPF

$$\phi_n(s) = \phi(s)\tau s/(1+\tau s) \qquad (17)$$

From the equations (16) and (17),

$$T_x(s) = \frac{(-K_1'' + K_2''\tau s)}{1+\tau s}\,\phi(s) \qquad (18)$$

The modified system equations are

$$\begin{bmatrix} s^2+a_1 & b_1 s + \dfrac{K_1-K_2\tau s}{1+\tau s} \\ b_2 s & s^2+a_2 \end{bmatrix} \begin{bmatrix} \psi(s) \\ \phi(s) \end{bmatrix} = \begin{bmatrix} \psi_o s + \dot{\psi}_o + b_1\phi_o \\ \phi_o s + \dot{\phi}_o + b_2\psi_o \end{bmatrix} \quad (19)$$

where $K_1 = K_1''/I_x$ and $K_2 = K_2''/I_x$

The determinant of (19) is given as

$$\Delta(s) = \frac{\left\{\tau s^5+s^4+s^3\left[(a_1+a_2)\tau - b_1b_2\right]+s^2\left[a_1+a_2-b_1b_2+b_2K_2\tau\right] + s(a_1a_2\tau - b_2K_1)+a_1a_2\right\}}{(1+\tau s)} \quad (20)$$

The characteristic equation can be taken as,

$$s^5+s^4/\tau + s^3(a_1+a_2-b_1b_2)+s^2\left[\frac{a_1+a_2-b_1b_2}{\tau}+b_2K_2\right] + s(a_1a_2 - b_2K_1/\tau)+ a_1a_2/\tau = 0 \quad (21)$$

## 5-1 STABILITY ANALYSIS

Equation (21) can be written as

$$(s^2+\alpha_1 s+w_o'^2)s^2+\alpha_2 s+w_n^2)(s+\alpha_3) = 0 \quad (22)$$

Comparing the co-efficients of equations (21) and (22) we obtain,

$$\alpha_2 = w_n^2\,(b_1b_2-b_2K_2\tau+w_n^2)/(b_2K_1+w_n^4) \quad (23)$$

$$\alpha_3 = w_n^2(1/\tau-\alpha_2)+\sqrt{w_n^4(1/-\alpha_2)^2+4b_2} \quad (24)$$

$$\alpha_1 = 1/\tau-\alpha_1-\alpha_2 = -b_2K_1/w_n^2 \quad (25)$$

$$\text{Also } w_o^2 = a_1a_2/w_n^2 \quad (26)$$

Equations (23) to (25) show that $\alpha_1, \alpha_2$ and $\alpha_3$ are all positive, thus ensuring system stability for positive value of the feedback gains $K_1$ and $K_2$.

## 5.2 TIME CONSTANTS OF THE CONTROL SYSTEM

The characteristic equation (22) can now be used to determine the time constants of the two modes of oscillations viz., the orbital mode $\tau_o$ and the nutational mode, $\tau_n$. Equation (20) can now be rewritten as,

$$\Delta(s) = (s^2+\alpha_1 s+w_o^2)(s^2+\alpha_2 s+w_n^2) \quad (27)$$

Since $\alpha_3 = 1/\tau$ . Also, from (19)

$$\phi(s) = \phi_o s^3+\dot{\phi}_o s^2+(a_1\phi_o-b_2\dot{\psi}_o+b_1b_2\psi_o)s + a_1(\dot{\phi}_o+b_2\psi_o)/\Delta(s) \quad (28)$$

considering first the nutation motion, the term $(s^2+\alpha_2 s+w_n^2)$ has a pair of complex conjugate roots at nutational frequency. Hence, the time constant for nutation damping can be readily obtained as,

$$\tau_n = 2/\alpha_2$$

The term $(s^2+\alpha_1 s+w_o^2)$ represents an overdamped system i.e., it has two real roots. Hence, there are two time constants for the orbital mode component which are approximatly in the ratio of 10 : 1. The subsequent analysis is done to find out which of the two time constants dominates.

Inverse Laplace transformation of (28) gives

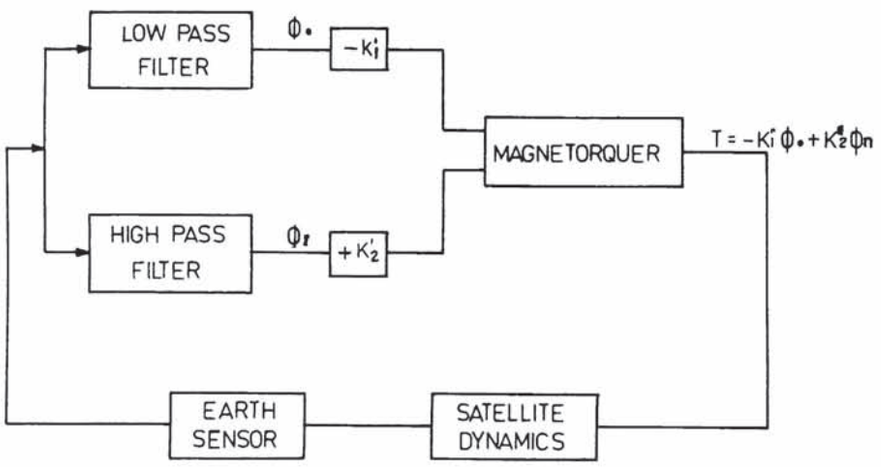



Fig.2 SCHEMATIC OF THE PROPOSED MAGNETIC CONTROLLER.

$$\phi(t) = \frac{1}{(\tau_2-\tau_1)}\left[e^{-\tau_1 t}(c_1\tau_1+c_2)-e^{-\tau_2 t}(c_1\tau_2+c_2)\right]+ e^{-\alpha_2 t/2}(c_3\cos W_n t + c_4 \sin W_n t) \quad (29)$$

where,

$$c_1 = -\left(\frac{b_1 b_2 \phi_o}{W_n^2} + \frac{\alpha_1 \dot{\phi}_o}{W_n^2} + \frac{b_2 \dot{\psi}_o}{W_n^2}\right) \quad (30)$$

$$c_2 = (a_1\dot{\phi}_o + a_1 b_2 \psi_o)/W_n^2 \quad (31)$$

$$c_3 = \phi_o\left(1+\frac{b_1 b_2}{W_n^2}\right) + \frac{\alpha_1\dot{\phi}_o}{W_n^2} + \frac{b_2\dot{\psi}_o}{W_n^2} \quad (32)$$

$$c_4 = \dot{\phi}_o - \frac{a_1 b_2 \alpha_2}{\alpha_1}\psi_o - \phi_o\alpha_1 \quad (33)$$

$$\tau_1 = (\alpha_1 + \sqrt{\alpha_1^2 - 4w_o^2})/2 \quad (34)$$

$$\tau_2 = (\alpha_1 - \sqrt{\alpha_1^2 - 4w_o^2})/2 \quad (35)$$

## 6.0 DESIGN CONSIDERATIONS

As seen from earlier analysis the system time constants depend upon the feedback gains $K_1$ and $K_2$. The magnitude of the gains is governed by factors such as maximum control torque and allowable pointing error.

The control torque produced by the magnetotorquer is limited due to the weak magnetic field at synchronous altitude, weight and power constraints. Also, the feedback gains should be selected so as to keep the satellite attitude error within allowable limits in the presence of disturbance torques. A conservative estimate of the control torque is arrived at by assuming a secular disturbance torque acting on the roll axis of the spacecraft. A value of $10^{-6}$ N.m is estimated as the constant disturbance torque for Apple spacecraft. The control torque should be about ten times greater than the disturbance torque and hence the controller should be capable of producing a torque of $10^{-5}$ N.M.

A realistic estimate of the required control torque can be obtained by considering the effect of Solar radiation torques on the satellite attitude. The solar radiation torques are cyclic in nature varying at orbital frequency. Assuming the

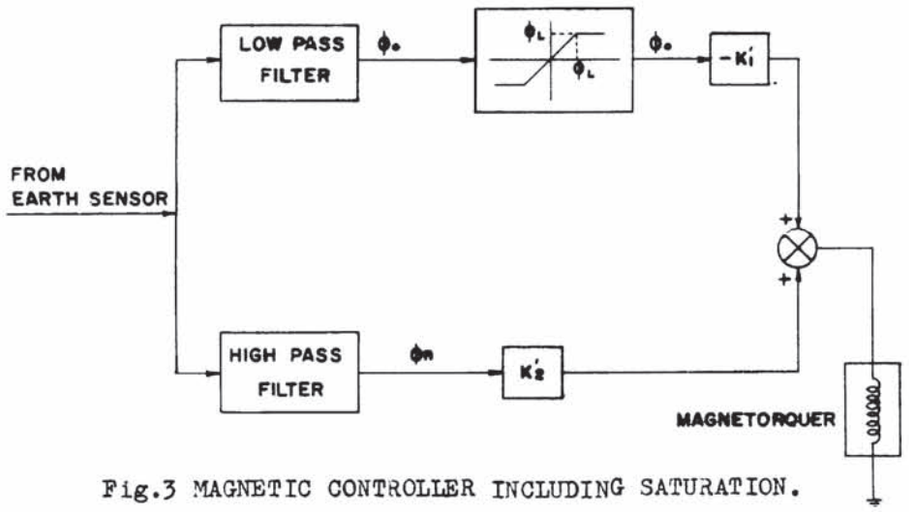



Fig.3 MAGNETIC CONTROLLER INCLUDING SATURATION.

It is obvious from (29) that both the time constants aid each other in reducing the error. However, the smaller time constant dominates over the other and hence the same can be used for design purposes.

$$\text{i.e., } \tau_o = 1/\tau_1 = 1/\tau_1 \quad (36)$$

solar torques $T_{XD}$ and $T_{YD}$ as

$$T_{XD} = A \sin w_o t$$

and $T_{YD} = B \cos w_o t$,

the system equations, assuming zero initial conditions can be written as,

$$\begin{bmatrix}\psi(s)\\ \phi(s)\end{bmatrix} = \frac{1}{\Delta(s)}\begin{bmatrix} s^2+a_2 & -b_1 s+\dfrac{K_1-K_2\tau s}{1+\tau s} \\ -b_2 s & s^2+a_1 \end{bmatrix}\begin{bmatrix} A'w_o/(s^2+w_o^2) \\ B's/(s^2+w_o^2)\end{bmatrix} \quad (37)$$

Where $A' = A/I_x$ and $B' = B/I_y$

Hence $$\phi(s) = \frac{B's^3+(B'a_1-A'w_o b_2)s}{\Delta(s)(s^2+w_o^2)} \quad (38)$$

our interest is to obtain an expression for the oscillatory component, '$\phi_{os}$' of the attitude error due to solar torques. Inverse Laplace transform of (38) yields

$$\phi_{os}(t) = \frac{B'w_o^2+A'w_o b_2-B'a_1}{\alpha_1 w_o(w_o^2\alpha_2^2+w_n^4)}(w_o\alpha_2 \cos w_o t - w_n^2 \sin w_o t) \quad (39)$$

The amplitude of the oscillation is given as $(B'w_o^2 + A'w_o b_2 - B'a_1)/\alpha_1 w_o w_n^2$.

If, '$\phi_m$', is the allowable error, then $\alpha_1$ should be so chosen to limit $\phi_{os}(t)$ to be less than $\phi_m$. Hence

$$\alpha_1 > (B'w_o^2+A'w_o b_2-B'a_1)/w_o w_n^2 \phi_m$$

substituting for $a_1$, $b_2 A'$ and $B'$,

$$\alpha_1 > (A+B)/h\phi m \quad (40)$$

Since

$$\alpha_1 \approx |b_2| K_1 |w_n^2 \quad (41)$$

$$K_1 > (A+B)/I_x \phi m \quad (42)$$

The magnetotorquer can be either air-cored or iron cored, and its detailed design is not dealt here. Iron cored torquers are preferred to air cored torquers since the former requires less power and weight. However, the iron cored torquers exhibit saturation characteristics and hence the controller should be designed such that the linearity of the feedback is maintained within desirable limits. If '$\phi_L$' is the error upto which proportional torque is maintained, the control torque should satisfy the condition,

$$T_x > (A+B)\phi_L/\phi_m \quad (43)$$

The constant $K_2(K_2'')$ can be determined by assuming a maximum half conning angle 'r' max. and equating $K_2''$ r max. with $T_x$. When both attitude and nutation are present, the output of the LPF is sent through a limiter shown in Fig.3. The output of the limiter is summed up with the nutational component and the combined signal can be used to drive the magnetotorquer. Also the required control law can be realised with filter alone, for example the LPF. Assuming $K_1 = K_2 = K^*$, the torque can be varied according to,

$$T = K^* (\phi - 2\phi_o) \quad (44)$$

Certain applications impose severe constraints on the long term attitude stability, for instance, a very High Resolution Radiometer(VHRR) in geosynchronous orbit demands long term stability of the order of $0.004^\circ$ for 30 minutes time for accurate prediction of the wind velocity. Such a requirement cannot be met by a normal momentum biased spacecraft. The proposed magnetic controller reduces the drift to a large extent. This can be practically reduced to zero for a unidirectional disturbance. Equation (39) gives the drift rate for a cyclic disturbance and the gain $K_1$ can be suitably selected to achieve the required drift rate.

### 6.1 BACK-UP ACTION WHEN GEOMAGNETIC FIELD REVERSES.

The magnetic control system loses its stability in the event of magnetic field reversal due to solar flares. Such phenomenon, as already mentioned in Section 2, occurs at the most once a year. The control system can be put out of action by ground commands after observing the roll behaviour. During this period the hydrazine gas jets may be used. The fuel consumption for this back-up action is negligible.

## 7.0 SIMULATION RESULTS

The proposed Control Scheme was simulated for a geosynchronous communication satellite, using a EAI 680 analog computer. The Linearised system equations were used. The relevant satellite and controller parameters are listed below:

Moments of Inertia

$I_x = 150$ Kgm$^2$
$I_y = 140$ Kgm$^2$
$I_z = 80$ Kgm$^2$

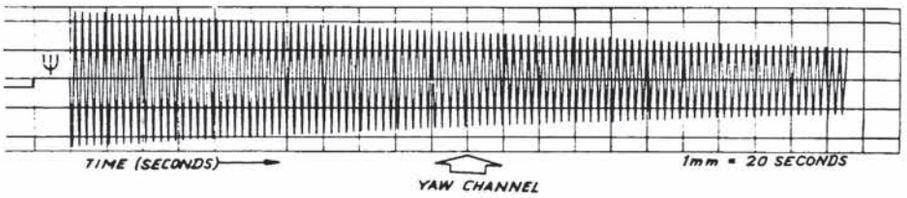
Nominal angular momentum of the wheel h = 22 N.M.S Feed back gains:
$K_1 = K_2 = 3.819 \times 10^{-5}$

Filter time constant = 100 sec.

$\phi_m = 0.1^\circ$
$r_m = 0.1^\circ$

The different cases studied are listed below.

APPLE MAGNETIC ATTITUDE CONTROL



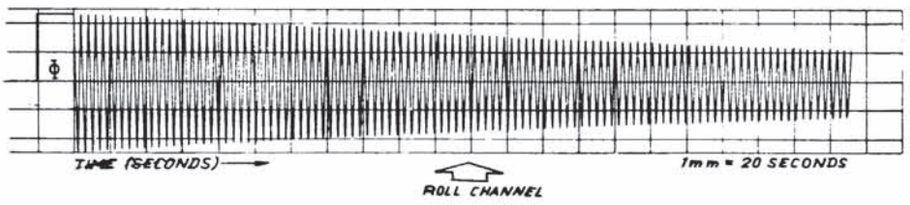
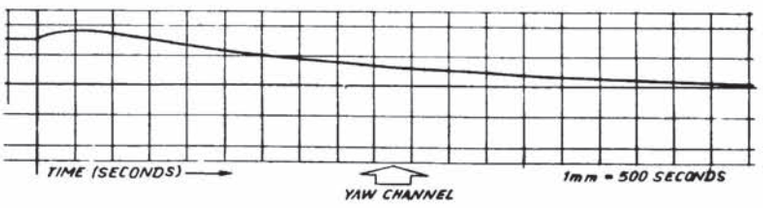




Fig.4 NUTATION DAMPING BY THE PROPOSED CONTROLLER.



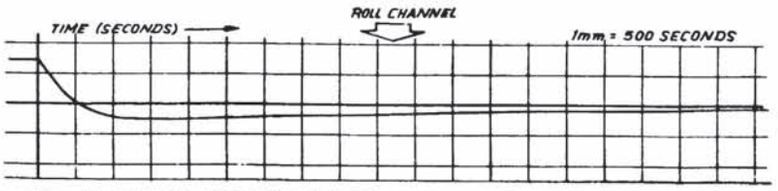




Fig.5 ERROR REDUCTION BY THE PROPOSED CONTROLLER.

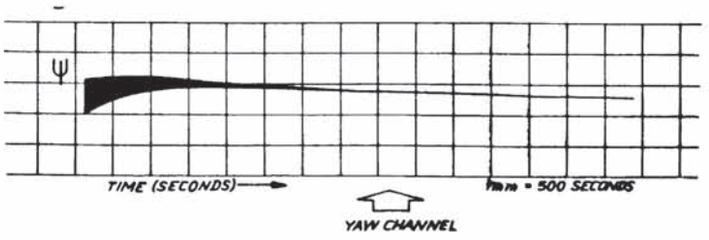
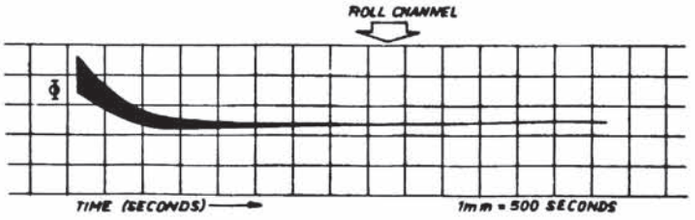



Fig.6 ERROR REDUCTION & NUTATION DAMPING (INCLUDING SATURATION)



1. Nutation damping by the proposed controller (Fig.4)
2. Error reduction by the proposed with Yaw and Roll errors including saturation characteristics (Fig.5)
3. Error reduction and nutation damping (Fig.6).

The time constants obtained from simulation plots matches well with the analytical expressions derived in section 5.

## 8.0 CONCLUSIONS

A new magnetic attitude control concept is proposed for momentum biased satellites in near equatorial orbits. Such a control concept is highly desirable for long life communication satellites. The closed loop control law provides error control as well as nutation damping. The linearised equations of motion are analysed to check for stability and to derive analytical expressions for system time constants and roll response. The proposed scheme was simulated using an EAI 680 analog computer for a typical communication satellite. The simulation results match well with the analytical results.

## ACKNOWLEDGEMENTS

The authors thank Prof.U.R. Rao, Director, ISRO Satellite Centre, for his keen interest and encouragement during the course of this work. The assistance rendered by the simulation group of Aeronautical Development Establishment, Bangalore, during the simulation of the Control scheme is duely acknowledged.

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